

Significance

This value is the force exerted by each of the four wheel cylinders. Note that we can add as many wheel cylinders as we wish. If each has a 2.50-cm diameter, each will exert 1.25×10^4 N. A simple hydraulic system, as an example of a simple machine, can increase force but cannot do more work than is done on it. Work is force times distance moved, and the wheel cylinder moves through a smaller distance than the pedal cylinder. Furthermore, the more wheels added, the smaller the distance each one moves. Many hydraulic systems—such as power brakes and those in bulldozers—have a motorized pump that actually does most of the work in the system.



14.3 Check Your Understanding Would a hydraulic press still operate properly if a gas is used instead of a liquid?

14.4 | Archimedes' Principle and Buoyancy

Learning Objectives

By the end of this section, you will be able to:

- Define buoyant force
- State Archimedes' principle
- Describe the relationship between density and Archimedes' principle

When placed in a fluid, some objects float due to a buoyant force. Where does this buoyant force come from? Why is it that some things float and others do not? Do objects that sink get any support at all from the fluid? Is your body buoyed by the atmosphere, or are only helium balloons affected (**Figure 14.19**)?



(a)



(b)



(c)

Figure 14.19 (a) Even objects that sink, like this anchor, are partly supported by water when submerged. (b) Submarines have adjustable density (ballast tanks) so that they may float or sink as desired. (c) Helium-filled balloons tug upward on their strings, demonstrating air's buoyant effect. (credit b: modification of work by Allied Navy; credit c: modification of work by "Crystl"/Flickr)

Answers to all these questions, and many others, are based on the fact that pressure increases with depth in a fluid. This means that the upward force on the bottom of an object in a fluid is greater than the downward force on top of the object. There is an upward force, or **buoyant force**, on any object in any fluid (**Figure 14.20**). If the buoyant force is greater than the object's weight, the object rises to the surface and floats. If the buoyant force is less than the object's weight, the object sinks. If the buoyant force equals the object's weight, the object can remain suspended at its present depth. The buoyant force is always present, whether the object floats, sinks, or is suspended in a fluid.

Buoyant Force

The buoyant force is the upward force on any object in any fluid.

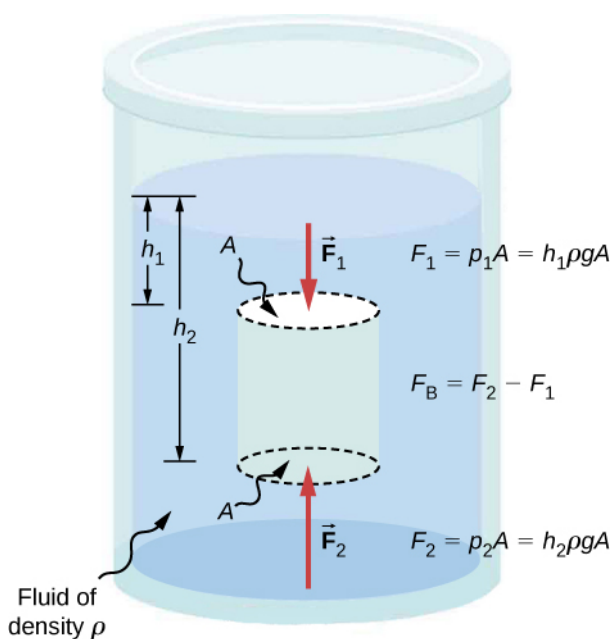


Figure 14.20 Pressure due to the weight of a fluid increases with depth because $p = h\rho g$. This change in pressure and associated upward force on the bottom of the cylinder are greater than the downward force on the top of the cylinder. The difference in the force results in the buoyant force F_B . (Horizontal forces cancel.)

Archimedes' Principle

Just how large a force is buoyant force? To answer this question, think about what happens when a submerged object is removed from a fluid, as in **Figure 14.21**. If the object were not in the fluid, the space the object occupied would be filled by fluid having a weight w_{fl} . This weight is supported by the surrounding fluid, so the buoyant force must equal w_{fl} , the weight of the fluid displaced by the object.

Archimedes' Principle

The buoyant force on an object equals the weight of the fluid it displaces. In equation form, **Archimedes' principle** is

$$F_B = w_{\text{fl}},$$

where F_B is the buoyant force and w_{fl} is the weight of the fluid displaced by the object.

This principle is named after the Greek mathematician and inventor Archimedes (ca. 287–212 BCE), who stated this principle long before concepts of force were well established.

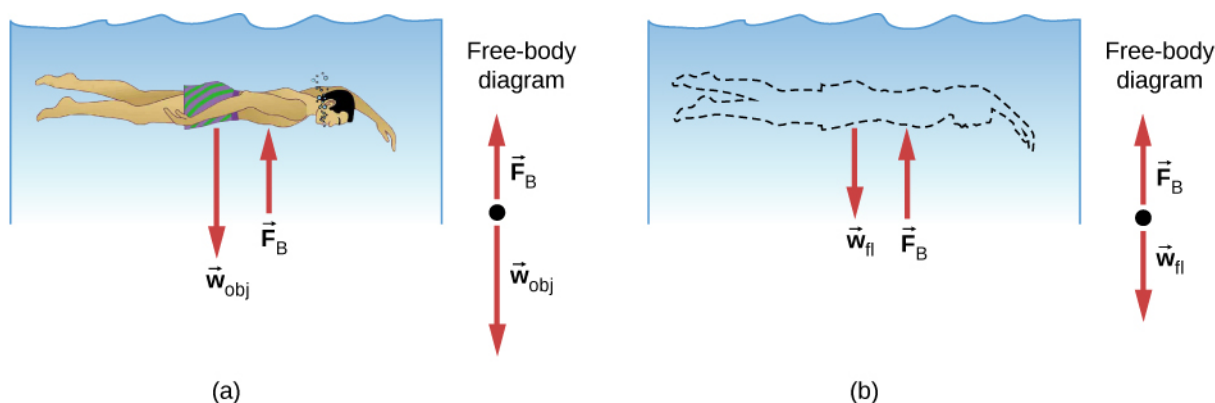


Figure 14.21 (a) An object submerged in a fluid experiences a buoyant force F_B . If F_B is greater than the weight of the object, the object rises. If F_B is less than the weight of the object, the object sinks. (b) If the object is removed, it is replaced by fluid having weight w_{fl} . Since this weight is supported by surrounding fluid, the buoyant force must equal the weight of the fluid displaced.

Archimedes' principle refers to the force of buoyancy that results when a body is submerged in a fluid, whether partially or wholly. The force that provides the pressure of a fluid acts on a body perpendicular to the surface of the body. In other words, the force due to the pressure at the bottom is pointed up, while at the top, the force due to the pressure is pointed down; the forces due to the pressures at the sides are pointing into the body.

Since the bottom of the body is at a greater depth than the top of the body, the pressure at the lower part of the body is higher than the pressure at the upper part, as shown in **Figure 14.20**. Therefore a net upward force acts on the body. This upward force is the force of buoyancy, or simply *buoyancy*.



The exclamation “Eureka” (meaning “I found it”) has often been credited to Archimedes as he made the discovery that would lead to Archimedes' principle. Some say it all started in a bathtub. To hear this story, watch this [video](https://openstaxcollege.org//21archNASA) (<https://openstaxcollege.org//21archNASA>) or explore **Scientific American** (<https://openstaxcollege.org//21archsciamer>) to learn more.

Density and Archimedes' Principle

If you drop a lump of clay in water, it will sink. But if you mold the same lump of clay into the shape of a boat, it will float. Because of its shape, the clay boat displaces more water than the lump and experiences a greater buoyant force, even though its mass is the same. The same is true of steel ships.

The average density of an object is what ultimately determines whether it floats. If an object's average density is less than that of the surrounding fluid, it will float. The reason is that the fluid, having a higher density, contains more mass and hence more weight in the same volume. The buoyant force, which equals the weight of the fluid displaced, is thus greater than the weight of the object. Likewise, an object denser than the fluid will sink.

The extent to which a floating object is submerged depends on how the object's density compares to the density of the fluid. In **Figure 14.22**, for example, the unloaded ship has a lower density and less of it is submerged compared with the same ship when loaded. We can derive a quantitative expression for the fraction submerged by considering density. The fraction submerged is the ratio of the volume submerged to the volume of the object, or

$$\text{fraction submerged} = \frac{V_{\text{sub}}}{V_{\text{obj}}} = \frac{V_{\text{fl}}}{V_{\text{obj}}}.$$

The volume submerged equals the volume of fluid displaced, which we call V_{fl} . Now we can obtain the relationship between the densities by substituting $\rho = \frac{m}{V}$ into the expression. This gives

$$\frac{V_{\text{fl}}}{V_{\text{obj}}} = \frac{m_{\text{fl}}/\rho_{\text{fl}}}{m_{\text{obj}}/\rho_{\text{obj}}},$$

where ρ_{obj} is the average density of the object and ρ_{fl} is the density of the fluid. Since the object floats, its mass and that

of the displaced fluid are equal, so they cancel from the equation, leaving

$$\text{fraction submerged} = \frac{\rho_{\text{obj}}}{\rho_{\text{fl}}}.$$

We can use this relationship to measure densities.

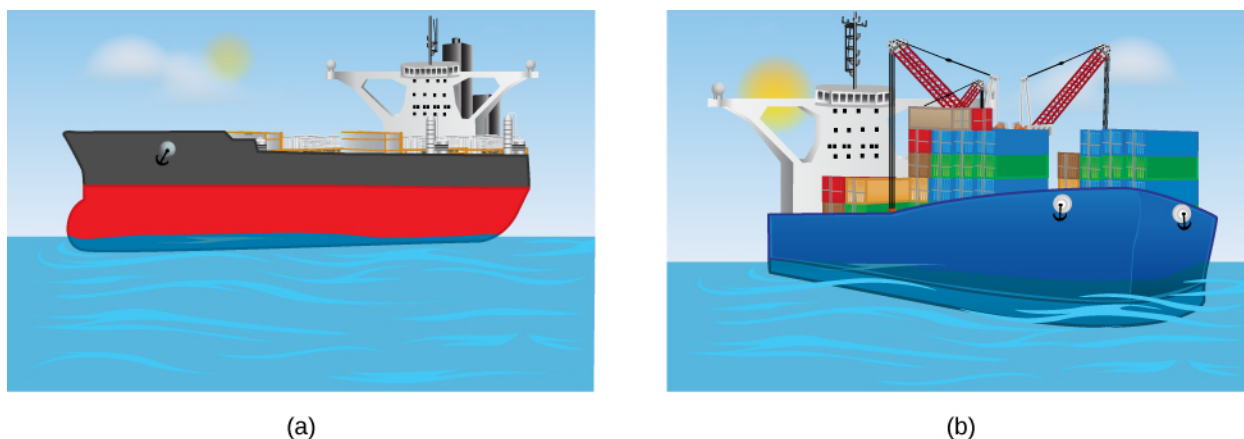


Figure 14.22 An unloaded ship (a) floats higher in the water than a loaded ship (b).

Example 14.4

Calculating Average Density

Suppose a 60.0-kg woman floats in fresh water with 97.0% of her volume submerged when her lungs are full of air. What is her average density?

Strategy

We can find the woman's density by solving the equation

$$\text{fraction submerged} = \frac{\rho_{\text{obj}}}{\rho_{\text{fl}}}$$

for the density of the object. This yields

$$\rho_{\text{obj}} = \rho_{\text{person}} = (\text{fraction submerged}) \cdot \rho_{\text{fl}}.$$

We know both the fraction submerged and the density of water, so we can calculate the woman's density.

Solution

Entering the known values into the expression for her density, we obtain

$$\rho_{\text{person}} = 0.970 \cdot \left(10^3 \frac{\text{kg}}{\text{m}^3}\right) = 970 \frac{\text{kg}}{\text{m}^3}.$$

Significance

The woman's density is less than the fluid density. We expect this because she floats.

Numerous lower-density objects or substances float in higher-density fluids: oil on water, a hot-air balloon in the atmosphere, a bit of cork in wine, an iceberg in salt water, and hot wax in a “lava lamp,” to name a few. A less obvious example is mountain ranges floating on the higher-density crust and mantle beneath them. Even seemingly solid Earth has fluid characteristics.

Measuring Density

One of the most common techniques for determining density is shown in **Figure 14.23**.

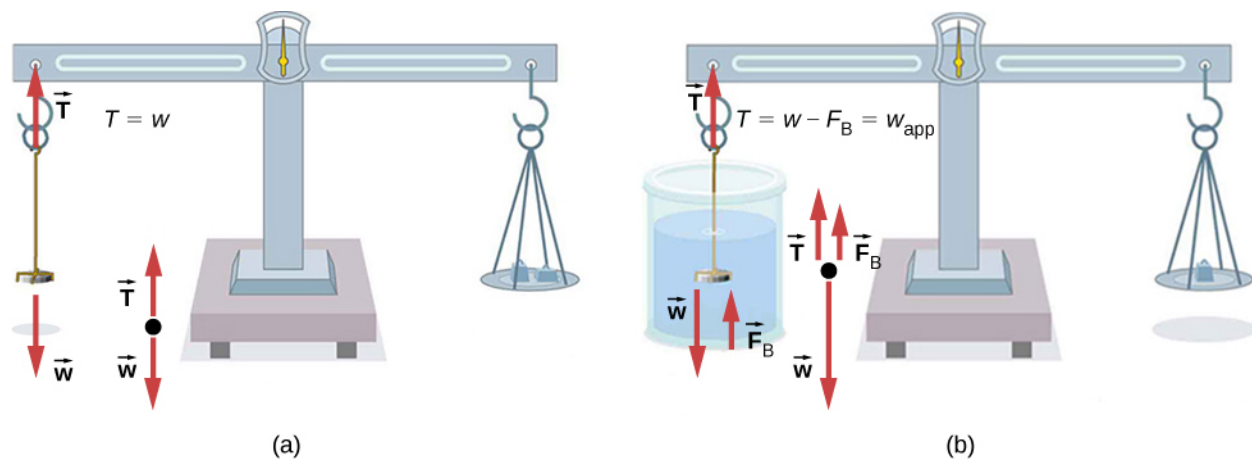


Figure 14.23 (a) A coin is weighed in air. (b) The apparent weight of the coin is determined while it is completely submerged in a fluid of known density. These two measurements are used to calculate the density of the coin.

An object, here a coin, is weighed in air and then weighed again while submerged in a liquid. The density of the coin, an indication of its authenticity, can be calculated if the fluid density is known. We can use this same technique to determine the density of the fluid if the density of the coin is known.

All of these calculations are based on Archimedes' principle, which states that the buoyant force on the object equals the weight of the fluid displaced. This, in turn, means that the object appears to weigh less when submerged; we call this measurement the object's apparent weight. The object suffers an apparent weight loss equal to the weight of the fluid displaced. Alternatively, on balances that measure mass, the object suffers an apparent mass loss equal to the mass of fluid displaced. That is, apparent weight loss equals weight of fluid displaced, or apparent mass loss equals mass of fluid displaced.

14.5 | Fluid Dynamics

Learning Objectives

By the end of this section, you will be able to:

- Describe the characteristics of flow
- Calculate flow rate
- Describe the relationship between flow rate and velocity
- Explain the consequences of the equation of continuity to the conservation of mass

The first part of this chapter dealt with fluid statics, the study of fluids at rest. The rest of this chapter deals with fluid dynamics, the study of fluids in motion. Even the most basic forms of fluid motion can be quite complex. For this reason, we limit our investigation to **ideal fluids** in many of the examples. An ideal fluid is a fluid with negligible **viscosity**. Viscosity is a measure of the internal friction in a fluid; we examine it in more detail in **Viscosity and Turbulence**. In a few examples, we examine an incompressible fluid—one for which an extremely large force is required to change the volume—since the density in an incompressible fluid is constant throughout.

Characteristics of Flow

Velocity vectors are often used to illustrate fluid motion in applications like meteorology. For example, wind—the fluid motion of air in the atmosphere—can be represented by vectors indicating the speed and direction of the wind at any given point on a map. **Figure 14.24** shows velocity vectors describing the winds during Hurricane Arthur in 2014.